Spline polynomials of the first kind

C.C. Lalescu

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1 Introduction

Spline polynomials of the first kind $\alpha_0^{(n,l)}$, as constructed for high order Hermite splines (by solving the equations for the splines), up to $n = 19$.

it should be easy to prove that

$$\alpha_1^{(n,l)}(x) = (-1)^l \alpha_0^{(n,l)}(1-x) \tag{1}$$

so only the α_0-s are given.

$$\alpha_0^{(3,0)}(x) = (x-1)^2\ (2\,x+1) \tag{2}$$

$$\alpha_0^{(3,1)}(x) = (x-1)^2\,x \tag{3}$$

$$\alpha_0^{(5,0)}(x) = -(x-1)^3\ \left(6\,x^2+3\,x+1\right) \tag{4}$$

$$\alpha_0^{(5,1)}(x) = -(x-1)^3\,x\ (3\,x+1) \tag{5}$$

$$\alpha_0^{(5,2)}(x) = -\frac{(x-1)^3\,x^2}{2} \tag{6}$$

$$\alpha_0^{(7,0)}(x) = (x-1)^4\ \left(20\,x^3+10\,x^2+4\,x+1\right) \tag{7}$$

$$\alpha_0^{(7,1)}(x) = (x-1)^4\,x\ \left(10\,x^2+4\,x+1\right) \tag{8}$$

$$\alpha_0^{(7,2)}(x) = \frac{(x-1)^4\,x^2\ (4\,x+1)}{2} \tag{9}$$

$$\alpha_0^{(7,3)}(x) = \frac{(x-1)^4\,x^3}{6} \tag{10}$$

$$\alpha_0^{(9,0)}(x) = -(x-1)^5\ \left(70\,x^4+35\,x^3+15\,x^2+5\,x+1\right) \tag{11}$$
$$\alpha_0^{(9,1)}(x) = -(x-1)^5\,x\ \left(35\,x^3+15\,x^2+5\,x+1\right) \tag{12}$$
$$\alpha_0^{(9,2)}(x) = -\frac{(x-1)^5\,x^2\ \left(15\,x^2+5\,x+1\right)}{2} \tag{13}$$
$$\alpha_0^{(9,3)}(x) = -\frac{(x-1)^5\,x^3\ (5\,x+1)}{6} \tag{14}$$
$$\alpha_0^{(9,4)}(x) = -\frac{(x-1)^5\,x^4}{24} \tag{15}$$

$$\alpha_0^{(11,0)}(x) = (x-1)^6\ \left(252\,x^5+126\,x^4+56\,x^3+21\,x^2+6\,x+1\right) \tag{16}$$
$$\alpha_0^{(11,1)}(x) = (x-1)^6\,x\ \left(126\,x^4+56\,x^3+21\,x^2+6\,x+1\right) \tag{17}$$
$$\alpha_0^{(11,2)}(x) = \frac{(x-1)^6\,x^2\ \left(56\,x^3+21\,x^2+6\,x+1\right)}{2} \tag{18}$$
$$\alpha_0^{(11,3)}(x) = \frac{(x-1)^6\,x^3\ \left(21\,x^2+6\,x+1\right)}{6} \tag{19}$$
$$\alpha_0^{(11,4)}(x) = \frac{(x-1)^6\,x^4\ (6\,x+1)}{24} \tag{20}$$
$$\alpha_0^{(11,5)}(x) = \frac{(x-1)^6\,x^5}{120} \tag{21}$$

$$\alpha_0^{(13,0)}(x) = -(x-1)^7\ \left(924\,x^6+462\,x^5+210\,x^4+84\,x^3+28\,x^2+7\,x+1\right) \tag{22}$$
$$\alpha_0^{(13,1)}(x) = -(x-1)^7\,x\ \left(462\,x^5+210\,x^4+84\,x^3+28\,x^2+7\,x+1\right) \tag{23}$$
$$\alpha_0^{(13,2)}(x) = -\frac{(x-1)^7\,x^2\ \left(210\,x^4+84\,x^3+28\,x^2+7\,x+1\right)}{2} \tag{24}$$
$$\alpha_0^{(13,3)}(x) = -\frac{(x-1)^7\,x^3\ \left(84\,x^3+28\,x^2+7\,x+1\right)}{6} \tag{25}$$
$$\alpha_0^{(13,4)}(x) = -\frac{(x-1)^7\,x^4\ \left(28\,x^2+7\,x+1\right)}{24} \tag{26}$$
$$\alpha_0^{(13,5)}(x) = -\frac{(x-1)^7\,x^5\ (7\,x+1)}{120} \tag{27}$$
$$\alpha_0^{(13,6)}(x) = -\frac{(x-1)^7\,x^6}{720} \tag{28}$$

$$\alpha_0^{(15,0)}(x) = (x-1)^8\ \left(3432\,x^7 + 1716\,x^6 + 792\,x^5 + 330\,x^4 + 120\,x^3 + 36\,x^2 + 8\,x + 1\right) \tag{29}$$

$$\alpha_0^{(15,1)}(x) = (x-1)^8\,x\ \left(1716\,x^6 + 792\,x^5 + 330\,x^4 + 120\,x^3 + 36\,x^2 + 8\,x + 1\right) \tag{30}$$

$$\alpha_0^{(15,2)}(x) = \frac{(x-1)^8\,x^2\ \left(792\,x^5 + 330\,x^4 + 120\,x^3 + 36\,x^2 + 8\,x + 1\right)}{2} \tag{31}$$

$$\alpha_0^{(15,3)}(x) = \frac{(x-1)^8\,x^3\ \left(330\,x^4 + 120\,x^3 + 36\,x^2 + 8\,x + 1\right)}{6} \tag{32}$$

$$\alpha_0^{(15,4)}(x) = \frac{(x-1)^8\,x^4\ \left(120\,x^3 + 36\,x^2 + 8\,x + 1\right)}{24} \tag{33}$$

$$\alpha_0^{(15,5)}(x) = \frac{(x-1)^8\,x^5\ \left(36\,x^2 + 8\,x + 1\right)}{120} \tag{34}$$

$$\alpha_0^{(15,6)}(x) = \frac{(x-1)^8\,x^6\ (8\,x + 1)}{720} \tag{35}$$

$$\alpha_0^{(15,7)}(x) = \frac{(x-1)^8\,x^7}{5040} \tag{36}$$

$$\alpha_0^{(17,0)}(x) = -(x-1)^9\ \left(12870\,x^8 + 6435\,x^7 + 3003\,x^6 + 1287\,x^5 + 495\,x^4 + 165\,x^3 + 45\,x^2 + 9\,x + 1\right) \tag{37}$$

$$\alpha_0^{(17,1)}(x) = -(x-1)^9\,x\ \left(6435\,x^7 + 3003\,x^6 + 1287\,x^5 + 495\,x^4 + 165\,x^3 + 45\,x^2 + 9\,x + 1\right) \tag{38}$$

$$\alpha_0^{(17,2)}(x) = -\frac{(x-1)^9\,x^2\ \left(3003\,x^6 + 1287\,x^5 + 495\,x^4 + 165\,x^3 + 45\,x^2 + 9\,x + 1\right)}{2} \tag{39}$$

$$\alpha_0^{(17,3)}(x) = -\frac{(x-1)^9\,x^3\ \left(1287\,x^5 + 495\,x^4 + 165\,x^3 + 45\,x^2 + 9\,x + 1\right)}{6} \tag{40}$$

$$\alpha_0^{(17,4)}(x) = -\frac{(x-1)^9\,x^4\ \left(495\,x^4 + 165\,x^3 + 45\,x^2 + 9\,x + 1\right)}{24} \tag{41}$$

$$\alpha_0^{(17,5)}(x) = -\frac{(x-1)^9\,x^5\ \left(165\,x^3 + 45\,x^2 + 9\,x + 1\right)}{120} \tag{42}$$

$$\alpha_0^{(17,6)}(x) = -\frac{(x-1)^9\,x^6\ \left(45\,x^2 + 9\,x + 1\right)}{720} \tag{43}$$

$$\alpha_0^{(17,7)}(x) = -\frac{(x-1)^9\,x^7\ (9\,x + 1)}{5040} \tag{44}$$

$$\alpha_0^{(17,8)}(x) = -\frac{(x-1)^9\,x^8}{40320} \tag{45}$$

$$\alpha_0^{(19,0)}(x) = (x-1)^{10} \left(48620\,x^9 + 24310\,x^8 + 11440\,x^7 + 5005\,x^6 + 2002\,x^5 + 715\,x^4 + 220\,x^3 + 55\,x^2 + 10\,x + 1\right) \tag{46}$$

$$\alpha_0^{(19,1)}(x) = (x-1)^{10}\,x \left(24310\,x^8 + 11440\,x^7 + 5005\,x^6 + 2002\,x^5 + 715\,x^4 + 220\,x^3 + 55\,x^2 + 10\,x + 1\right) \tag{47}$$

$$\alpha_0^{(19,2)}(x) = \frac{(x-1)^{10}\,x^2 \left(11440\,x^7 + 5005\,x^6 + 2002\,x^5 + 715\,x^4 + 220\,x^3 + 55\,x^2 + 10\,x + 1\right)}{2} \tag{48}$$

$$\alpha_0^{(19,3)}(x) = \frac{(x-1)^{10}\,x^3 \left(5005\,x^6 + 2002\,x^5 + 715\,x^4 + 220\,x^3 + 55\,x^2 + 10\,x + 1\right)}{6} \tag{49}$$

$$\alpha_0^{(19,4)}(x) = \frac{(x-1)^{10}\,x^4 \left(2002\,x^5 + 715\,x^4 + 220\,x^3 + 55\,x^2 + 10\,x + 1\right)}{24} \tag{50}$$

$$\alpha_0^{(19,5)}(x) = \frac{(x-1)^{10}\,x^5 \left(715\,x^4 + 220\,x^3 + 55\,x^2 + 10\,x + 1\right)}{120} \tag{51}$$

$$\alpha_0^{(19,6)}(x) = \frac{(x-1)^{10}\,x^6 \left(220\,x^3 + 55\,x^2 + 10\,x + 1\right)}{720} \tag{52}$$

$$\alpha_0^{(19,7)}(x) = \frac{(x-1)^{10}\,x^7 \left(55\,x^2 + 10\,x + 1\right)}{5040} \tag{53}$$

$$\alpha_0^{(19,8)}(x) = \frac{(x-1)^{10}\,x^8\ (10\,x + 1)}{40320} \tag{54}$$

$$\alpha_0^{(19,9)}(x) = \frac{(x-1)^{10}\,x^9}{362880} \tag{55}$$

From the form of the polynomials it seems we can use the series given at `http://www.research.att.com/~njas/sequences/A046899`:

$$\alpha_0^{(n,l)}(x) = \frac{(1-x)^{m+1}}{m!}\frac{x^l}{l!}\sum_{k=0}^{m-l}(m+k)!\frac{x^k}{k!} \tag{56}$$